Progressive deformation of elliptical markers at the hinge zone of similar fold

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Abstract-Partitioning of the progressive deformation that takes place during the development of a similar fold gives some momentary, infinitesimal episodes of compressional and extensional regime. An initial heterogeneous strain field may be superimposed by a homogeneous deformation and this may be followed by a rotatory shear strain. Episodic or incremental strain may change the neutral points to ellipse and these ellipses may be transposed later by heterogeneous strains. Neutral points or reciprocal strain ellipses are auxiliary circles that may be formed even when one of the axes of the ellipse does not change its lengths. Infinitesimal strains together constitute the finite strain ellipses. In an ideal situation the angular displacement of the minor axis of the ellipse helps us to determine the coordinates of the initial reference frame. Familiar geometrical relationships are applied to describe the episodic differential stress fields on an elliptical marker at the hinge zone of a similar fold during progressive deformation.

Keywords: Neutral point; auxiliary circle; strain partitioning; elliptical marker, momentary strain, similar fold, progressive deformation

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1. INTRODUCTION

Heterogeneous strain fields contain some neutral points (or isotropic points), where strain in all directions is zero [1]. These are the points where the finite strain trajectories as in the case of simple shear similar fold changes from e_l to e_s (e_l and e_s stands for principal long axis and short axis integration of strain ellipses). However, it would be possible to obtain neutral points at the hinge zone of the other folds whose internal strain does not differ. Pattern of strain trajectories around isotropic points help in distinguishing them from other points of the strain fields [2] .Neutral points also known as reciprocal strain ellipse, is an ellipse before deformation and transforms into a circle with unit radius after deformation.

Considerable variation in finite strain is indicated during the various stages of progressive deformations leading to the development of a similar fold structure. A mechanism for development of ideal similar fold is analyzed. It is observed that the hinge zone (where the neutral points occur) of a fold structure (of constant thickness) strain states may vary from homogeneous to heterogeneous and rotatory movement may also be associated with it causing further deformation of the neutral points. Initial circles representing the neutral points at the hinge zone of a similar fold may become elliptical if the fold structure is acted on by a homogeneous shear component (Fig.7a and 8b). If a subsequent rotatory component is added then the ellipses will rotate around a vertical axis and aspect ratio will also be increased. Ideal finite strain state represented in Fig.7b, indicate that length of the minor axis of the ellipse can be calculated by employing the equation derived and the angular shear for these ellipses can be determined.

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2. GENERAL LINEAR TRANSFORMATION

Coordinates of a point in deformed condition are functions of the undeformed coordinates [3]. To illustrate the transformation of an ellipse to a circular from, let us start with the Lagrangian equation for linear coordinate transformation given by-

$$x' = ax + by ....(1)$$

$$y' = cx + dy ....(2)$$

or, the strain matrix

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Where, a, b, c and d are constants (Fig.1)

If u and v are vector components defining the displacement vector field given by

$$u = f_1(x', y') - x \text{ and } v = f_2(x' - y') - y$$

The displacement vector gradient will be

$$\begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial x'} & \frac{\partial v}{\partial y'} \end{pmatrix}$$

For a homogeneous body translation or rotation, all terms in the above matrix must be constant

Now, if x′ and y′ are the coordinates of a point on a circle with unit radius, then the equation of the circle would be –

$$x'^2 + y'^2 = 1$$

employing the values of x′ and y′ from the eqs. ((1) and (2), we get

$$(ax+by)^2+(cx+dy)^2=1$$

$$or\ (a^2+c^2)x^2+(b^2+c^2)y^2+2(ab+cd)xy=1$$

$$or\ (a^2+c^2)x^2+2(ab+cd)xy+(b^2+c^2)y^2=1......(3)$$

eqn. (3) is a special form of the ellipse centered at origin.

Now, if in the above eqn. (3), $(a^2+c^2)=1,\ (ab+cd)=0$ and $(b^2+c^2)=1.$, the eqn. (3) will become $x^2+y^2=1$ which is the equation of a circle of unit radius.

We shall try to illustrate that the standard equation of an ellipse centered at origin can become an equation of a circle.

Let us take the equation as

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

In Fig.2, C is taken as the origin and let $CQ=a$

The equation of the circle is $x^2+y^2=a^2$

Now, if the ordinate NP is extended to meet the circle at the point Q, we can have the two points P and Q with the coordinates (x′, y′) and (x″, y″). The two points, P on the ellipse and Q on its' auxiliary circle, are called corresponding points.

Since P is on the ellipse,

Therefore $$\frac{x'^2}{a^2}+\frac{y'^2}{b^2}=1..........(4)$$

Now, if this equation becomes a circle where Q is a point

We have, $$\frac{x'^2}{a^2}+\frac{y''^2}{a^2}=1..........(5)$$

From the eqs. (4) and (5) , we have,

$$\frac{y'^2}{b^2}=\frac{y''^2}{a^2}$$

$$or\ \frac{y'}{y''}=\frac{b}{a}$$

$$or\ \frac{PN}{QN}=\frac{b}{a}$$

$$or PN=\frac{b}{a}QN.........(6)$$

Therefore, if ordinate PN is extended by an amount (b/a) to the point Q upon a fixed diameter, then the locus of Q defines a circle. This auxiliary circle is a reciprocal strain ellipse. In Fig.3, states of strain during a steady motion progressive pure shear, is illustrated. Both in Figs 3a and b material lines are oriented at regular intervals and each is labeled with an angle it originally makes with X_1direction. These lines are rotated anticlockwise (upper half) and clockwise (lower half) with the progressive

deformation of the ellipse to a circle of unit radius. As a result, points P_1, P_2, moves to Q_1, Q_2, …and points P'_1,P'_2, move to Q'_1, Q'_2, ……At each strain increment, the same material lines will remain as principal strain axis describing a progressive coaxial deformation [4]. In three dimensions this is considered as an ideal strain and a simple extension [5].where all points move parallel to a straight line, if we consider the straight line to be the Y axis. We also have $e_y=ky$ and $e_x=e_z=0$ where k will take different values from one end of the diameter to the other end.

(∈ = epsilon)

Let us also consider that the non zero elongation $\in_3$ takes a magnitude of 0.5 before deformation, then the displacement equations can be given [6]for uniaxial extension are:

$$u_1=0X_1+0X_2+0X_3$$
$$u_2=0X_1+0X_2+0X_3$$
$$u_3=0X_1+0X_2+0.5X_3$$
$$u_1=0x_1+0x_2+0x_3$$
$$u_2=0x_1+0x_2+0x_3$$
$$u_3=0x_1+0x_2+1x_3$$
$$(where, x_3=2X_3)$$

3. VOLUME CHANGE

Suppose, before deformation the elipticity of the ellipse R= 1.05 and OY= OY′ = 0.25 (Figs 5(a) and (b)). After deformation (pure shear) YY′ =XX′, so that OY and OY′ has to increase 25 % to become a circle of unit radius. A line making an angle of 0° with OX direction and having a length of 0.5 will finally make 90° with OX and will coincide with OY .In the process 0.25 length will add to OY and the line will subtend a curvature of π/2 and the foot of the projection line will move from X to O as the point P takes an anticlockwise rotation from X to Y (Fig .5 c). The movement of p from X to Y, covering an azimuth of π/2 gives a linear vertical upliftment of the point O to Y (Fig.5d) with an increment of 25% on either directions (i.e. OY and OY′).

Quantification of the various stages of progressive deformation leading to the development of a similar fold indicates considerable finite strain variation. It is observed that if the fold is formed by heterogeneous simple shear with constant T and λ=1, the differently oriented strain ellipses have direction of no finite longitudinal strain parallel to the axial surfaces of the fold [7]. Near the hinge zone of a similar fold, the ellipses are as represented by the strain marker in the Fig.6. At this infinitesimal incremental stage, if the ellipse gets an extension along the minor axis direction (Figs. 6 (a) and (b)) it may become a unit circle upon a fixed diameter, which may be taken as of unit length.

In the Figs 6a and 6b, the line of no finite elongation for both the ellipses are parallel to the axial surface of the fold[7]. If the circles are auxiliary circles then the major axis of the ellipses will be the fixed diameter. This will take place under an extensional regime along the minor axes of the ellipses. Therefore, both the ellipses may become a circle or become reciprocal strain ellipses or neutral points at the hinge zone (Fig 6 c). This could well be considered as an ideal situation. Any deviation from this condition such as heterogeneous simple shear with a homogeneous strain will produce ellipses (instead of circles or neutral points at the hinge zone) with high ellipticity (R) and a quadratic elongation of λ for the axial plane parallel layer thickness T for the similar fold[7]. A component of shear acting across the layer may further rotate the elliptical markers in addition to the transposition of the layers as passive markers. Moreover, if we consider that the incremental strain axis coincides with the finite strain axis (considering coaxial strain accumulation), under the extensional regime along short axis direction, the zone of extension in Fig.7a should be exactly becoming the zone shortening and vice-versa. Therefore, we shall consider no internal vorticity as far as rotation of the material lines are concerned with zero rigid body rotation. Let the subsequent deformation initiates with a momentary compression. If this is followed by a shear (slightly layer oblique), further compression of the ellipse takes place (Fig.7a). Rotation of the semi minor axis of the ellipse may take a final position (indicating a finite longitudinal strain) making ψ_1 shear angle with the vertical axis.

4. ANALYSIS OF THE STRAIN STATES

The mechanism involved for the development of an ideal similar fold has never been explained satisfactorily. Shear planes are assumed to be acting across the layering and are considered as parallel to each other. A deviation from this condition will take place if, i) shear planes are not essentially parallel to each other ii) shear planes acting oblique to the layer due to the layer oblique shear iii) a layer parallel shear movement iv) flattening normal/oblique to the harmonically buckled multi layer.

In an ideal situation layer parallel strain decreases towards the middle of the each folded layer [8]. In Fig.8a, heterogeneous simple shear is applied in a layer of constant thickness and strain ellipses and the neutral points will be arranged as shown in the figure. In Fig. 8b, heterogeneous simple shear is associated with a homogeneous strain with constant displacement gradient. This caused the neutral points to become elliptical because of shear movement. Small discontinuities and displacements within the individual grains merge to give the overall picture of the ellipse continuity [1]. In Fig.8c shear planes acting obliquely (due to the momentary layer oblique shear) give rise to a situation where homogeneous simple shear will be associated with the situation depicted in Fig.8b. Fig.7a_represents a circle with center (a, o) as the reciprocal strain ellipse. Let ABCD be the initial square with coordinate of point B (x, y). Now, because of the homogeneous shear (see Fig. 8b) the circle will become an ellipse. In Fig.7a only the upper half of the ellipse is shown enclosed within the rectangle EFGH. Since the origin is taken at D, therefore, EH and CT represent the major and semi minor axes of the ellipse. Now, if the shearing is more, then the ellipticity will further increase. semi minor axis of the ellipse be$1+e_2$. In the Fig 7a, MNPD is the square to be formed in an ideal condition with the Let $1+e_1$ be the semi minor axis of the ellipse which will rotate vertically changing its' ellipticity further. Let the rotated coordinate of N (x_2 y_2). Let coordinate of R be (x_1 y_1). Let, minor axis of the rotated ellipse makes ψ_1 angle with CS (the original position of the ellipse i.e. at unrotated stage) and α angle with CP.

Now, from the Fig 7b, we have

$\tan\psi_1 = \dfrac{x_1 - x}{y_2}$ $x_1 = x_2$ The following observations are made:

$$a \quad 1+e_2{}^2 = x_1 - x{}^2 + y_2^2 \quad from\square CSN$$

$$or\ 1+e_2{}^2 = (y_2 \tan\psi_1)^2 + y^2{}_2$$

$$or\ 1+e_2{}^2 = y^2{}_2\ 1+\tan\psi_1{}^2$$

$$or(1+e_2)^2 = y^2{}_2 \sec^2\psi_1$$

$$or 1+e_2 = y_2 \sec\psi_1 \ldots\ldots\ldots\ldots\ 7$$

$$b \quad \tan\alpha = \tan(90^\circ - \psi_1)$$

$$\tan\alpha = \cot\psi_1$$

$$\tan\alpha = \frac{1}{\tan\psi_1}$$

$$or \tan\alpha \tan\psi_1 = 1 \ldots\ldots\ldots\ldots\ 8$$

From the above four equations following corollaries are made

1. In a terrain showing progressive deformation, if $\psi_1, \psi_2, \psi_3 \ldots\ldots\ldots$ etc. are measured then we have the CS direction. Now, if CQ = CS, then we can construct a circle taking CQ as diameter. Construction of the circle will give us the point D which is the origin such that the coordinates of C be (a, 0). From the origin coordinates of x, y, x_1,y_1 and x_2,y_2 can be measured.

From the relation

$$\tan\psi_1 = \frac{x_1 - x}{y_2}$$

$$(c) 1+e_2 = y_2 \ldots\ldots\ldots\ldots\ 9$$

$$d \quad 1+e_2{}^2 = 1+e_1{}^2\ Sec^2\psi_1$$

$$or\ 1+e_2 = 1+e_1\ Sec\psi_1$$

$$or Sec\psi_1 = \frac{1+e_2}{1+e_1} \ldots\ldots\ldots\ldots\ 10$$

if the coordinates are known then tan ψ_1 can be determined. If ψ_1 is measured, then from tan ψ_1, coordinate position can be checked whether correct or wrong.

2. Using the above value of ψ_1 and the known value of y_2 we can find e_2 from equation (7) and the value of e_1 from equation (9)

3. With the above value of $\psi_{1,}$ we can also determine the value of α from the equation (8)

4. The above equation will hold true if in the external coordinate reference frame does not change or changes homogeneously through an angular shear ψ_1 in the clockwise or anticlockwise direction.

5. In the ideal situation x will be half of x_1 and x_2. Since x corresponds to the radius and x_1 and x_2 corresponds to the diameter of the original circle,

Therefore, in an ideal condition,

$$\tan\psi_1 = \frac{x_1 - \frac{1}{2}x_1}{x_1} = \frac{1-\frac{1}{2}}{1} = \frac{1}{2}$$

$$\psi_1 = \tan^{-1}\frac{1}{2}.........\ 11$$

$$or\psi_1 < 90^0$$

6. If initially the reference frame is a square (say Fig 7b is the initial state), then the diagonal of the square will make 45^o with the base. Hence minor axis (CN in Fig.7b) will make more than 45^o with the base and as a result ψ_1 will be less then 45^{o}gradually towards right, which is in accordance with what is obtained in equation (11).

7. When the ellipse becomes the auxiliary circle, ¼πab area becomes ¼πa^2

Therefore, increase in area

$$=\frac{1}{4}\pi a(a-b)$$

$$=\frac{1}{4}\pi a(0.25b)$$

$$=\frac{1}{16}\pi ab$$

Or, an area of 1/16 π*ab* is increased in the original area of the ellipse, i.e. ¼π*ab*

Therefore, when the ellipse has become the circle, 25% area is added. The movement of the point O to Y (Fig.5d) with the 25% increment of the line in both ways (i.e. OY and OY′) creates an increment of 25% in the area of the ellipse.

5. .CONCLUSIONS

Momentary extensional regime (perpendicular to the major axis of the ellipses) is responsible for transferring near hinge zone ellipses to circles at the hinge zone proper in the initial stages of the development of a similar fold. These are auxiliary circles (isotropic points) develop from the ellipses even when the length of the major axis remains unchanged. Superposition of a layer parallel simple shear helped the circles to take the elliptical shape. This type of layer parallel movement, especially along hinge zone of a similar fold will produce ellipses whose major axis will be perpendicular to the axial trace of the fold. The picture of the movement can be visualized as shear movement along planes which are essentially parallel to the axial plane of the fold and also contain the axis of the fold. Superposition of a heterogeneous strain field at this movement may help in rotating the ellipse around a vertical axis. The axial plane will contain this vertical axis and if the axial plane becomes curviplanar then this ellipses will rotate as already mentioned.

The factorization of the strain depends on the choice of the origin [9]. It is also emphasized that identification of the elliptical markers and strain determination from these fabric are independent of the use of visible bedding or cleavage trace as the reference axis [10].

As proposed, in an ideal situation, the angular deviation of this ellipse axis and the length of the major and the minor axis of the finite ellipse will give us the coordinates of the initial reference frame (assuming a square) and most important, the coordinates of the origin of the reference frame can be determined. However, the above condition will hold true if the initial reference frame does not change or changes homogeneously through an angular shear in clockwise or anticlockwise direction.

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Figure captions

Fig.1. Unit square deformed into a parallelogram with length of sides given as x′ =ax + by and y′ = cx + dy

Fig.2.The auxiliary circle of an ellipse. The auxiliary circle is the circle described on the major axis AA´ as diameter

Fig.3. States of strain during a steady progressive pure shear. Material lines labeled as P_1 P_2 etc.(upper half) and P'_1 P'_2 etc. (lower half) making angle with X_1 in the undeformed state. As a result of deformation these points move to Q_1 ,Q_2 etc. (upper half) and Q'_1 ,Q'_2 etc. (lower half)

Fig.4. Pure shear (uniaxial extension).The non zero elongation is parallel to X_3= x_3 before deformation.
The magnitude of is taken as 0.5 before deformation.

Fig.5. Ellipse deforming to a circle (a) *XX′*= 1 unit, *YY* = 0.5 unit lengths respectively. (b) Material lines running anticlockwise (upper half) and clockwise (lower half). Arrowheads indicate particle motion during progressive deformation.(c) The foot of the projection lines from the points P_1 P_2 move towards ‘O’ on the X axis and away from ‘O’ on the Y axis during the anticlockwise rotation of P_1 to P_2. (d) Movements of the points from ‘O’ to Y concede an increment of 25% in length bothways of Y -axis.

Fig.6. (a) and (b) Lines of no finite elongation are parallel to the axial planes of the fold. (c) Reciprocal strain ellipse at the hinge zone due to pulsating extension perpendicular to the major axis of the ellipses.

Fig.7. (a) Reciprocal strain ellipse with (a, 0) as the center, at the hinge zone. Layer normal homogeneous shear produced the ellipse with EH as the major axis. Incremental homogeneous shear will further reduce the length of the major axis to IL as shown in the figure. Clockwise rotation of the ellipse through an angle ψ_1 due to the heterogeneous shear is shown. Note the position of Q in the first circle, deformed ellipse and in the rotated position where it is designated as N (x_2, y_2). (b) Reference frame for the final or initial position of the ellipse. CS and CN will assume lengths $1+e_1$ and $1+e_2$ respectively. In case of the final reference frame being a square, the semi minor axis of the ellipse (here CN) will always make less than 45^0(anticlockwise from the base) gradually towards right.

Fig.8 (a).Strain ellipses and neutral points at the hinge zone. Momentary expansion normal to the major axis of the ellipse is responsible for the development of neutral points. (b) Addition of a homogeneous strain deformed the neutral points to strain ellipses. (c) Momentary layer oblique shear rotates the ellipses. This is due to the addition of a heterogeneous strain as next incremental strain.

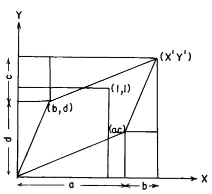

Fig. I

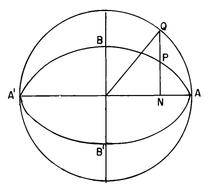

Fig. 2

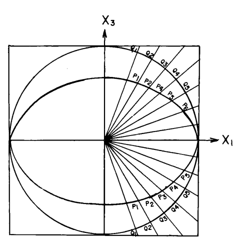

Fig. 3

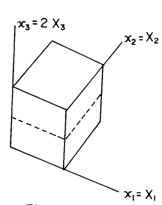

Fig.4

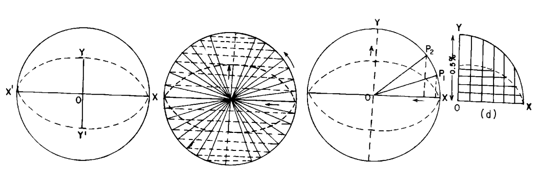

Fig.5

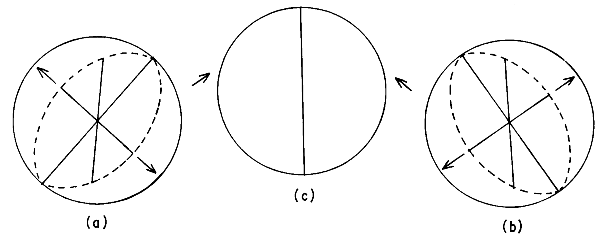

Fig.6

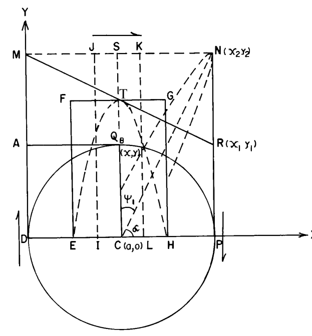

Fig. 7 (a)

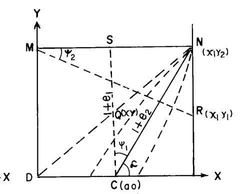

Fig. 7(b)

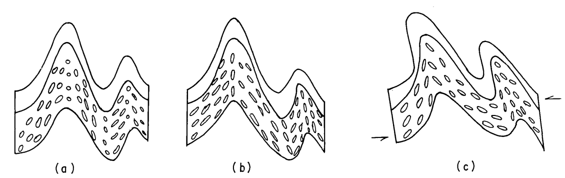

Fig.8